Retarded functional differential equations on manifolds: existence and bifurcations results

Pierluigi Benevieri
IME Universidade de São Paulo

We investigate T-periodic parametrized first and second order retarded functional equations on differentiable manifolds. We prove existence and global continuation results for T-periodic solutions. The approach is topological and is based on the degree theory for tangent vector fields as well as on the fixed point index theory.

Periodic solutions to perturbed Kepler problems

Alberto Boscaggin
University of Torino, Department of Mathematics

As well known (by third Kepler's law) the Kepler problem has many periodic solutions with minimal period T (for any given $T > 0$). We will try to understand how many of them survive after a T-periodic external perturbation preserving the Newtonian structure of the equation. In doing this, we will be naturally led to the concept of generalized solutions and to the theory of regularization of collisions in Celestial Mechanics.

Joint work with Rafael Ortega (Granada) and Lei Zhao (Augsburg).

Fast solutions and asymptotic behavior in reaction diffusion equations

Juan Campos
Universidad de Granada Spain

Let $f : [0,1] \to \mathbb{R}$ be C^1 and $f(0) = 0 = f(1)$, and consider the reaction diffusion

$$u_t = u_{xx} + f(u), t \geq 0, x \in \mathbb{R},$$

with initial datum $u(t,0) = u_0(x)$ with compact support, $0 \leq u_0(x) \leq 1$ and nontrivial.

When $f(u) > 0$ on $(0,1)$, it is known the existence of a value $c^* > 0$ with the property:

$$u(t, x + ct) \to 0 \text{ for any } x \in \mathbb{R} \text{ as } t \to \infty$$

holds for $|c| \leq c^*$. This number is connected with the speed of propagation of travelling waves.

When the stationary states of $\{\alpha \in [0,1] : f(\alpha) = 0\}$ are different of the extreme ones, the situation starts to be more complicate. However one can ask if there is a value $c^* > 0$ where the last property holds independently of u_0. This number is also connected with the speed of propagation of travelling waves, however only those of fast nature have to be consider.

[1] Arias, M.; Campos, J. Fast solutions and asymptotic behavior in a reaction-diffusion equation. J. Differential Equations 259 (2015) 54065439.

Uniform persistence in a prey-predator model with disease in one of the species

Tobia Dondè
Università degli Studi di Udine - Department of Mathematics, Computer Science and Physics
donde.tobia@spes.uniud.it

Following the theoretical approach to persistence mainly contained in [1] we give a formal explanation of the numerical results obtained in [2] on a certain predator-prey model with functional response of Holling type II equipped with a infectious disease in one of the two populations.

The proof relies on several repelling conditions that can be applied in turn on a suitable Morse decomposition of the boundary. It is shown how the basic reproduction number R_0 arises from these conditions and the crucial role of the stable limit cycle of the underlying infection-free model. Numerical experiments are carried out to display the change in the dynamics when some hypotheses fail.

Keywords: uniform persistence, infectious disease, basic reproduction number, prey-predator

[1] Josef Hofbauer. A unified approach to persistence. *Acta Applicandae Mathematicae*, 14(1-2):11–22, 1989.

[2] Andrew M. Bate and Frank M. Hilker. Predator–prey oscillations can shift when diseases become endemic. *Journal of Theoretical Biology*, 316:1–8, 2013.

Coincidence degree theory in product spaces and applications to cyclic feedback type systems

Guglielmo Feltrin
Università degli Studi di Torino
guglielmo.feltrin@unito.it

Using Mawhin's coincidence degree theory, we obtain some new continuation theorems which are designed to have as a natural application the study of the periodic problem for cyclic feedback type systems. Applications to vector ordinary differential equations with a ϕ-Laplacian operator will be discussed. Our main contribution in this direction is a continuation theorem for the periodic problem associated with $(\phi(u'))' + \lambda k(t, u, u') = 0$, under the only assumption that ϕ is a homeomorphism. This talk is based on a joint work with F. Zanolin (University of Udine).

Dynamical tools for the scalar curvature equation

Matteo Franca
Dipartimento di Scienze Matematiche, Università Politecnica delle Marche
Ancona - Italy.
email: franca@dipmat.univpm.it

In this talk we discuss the problem of existence and multiplicity of radial ground states with fast decay (GS for short) for

$$\Delta u + [1 + \epsilon k(|x|)]u^q = 0$$

where $x \in \mathbb{R}^n$, $n \geq 3$, $q = \frac{n+2}{n-2}$, $k \in C^1$. Such an equation arises naturally from differential geometry, but also finds applications as a model in quantum mechanics and astrophysics.

Assume that k has a unique critical point, say $k(1)$, then if it is a maximum under mild conditions it admits a unique GS, while if it is a minimum it admits a large number of GS if $\epsilon > 0$ is small enough. This latter result was proved in the celebrated paper **[1]**, which gave rise to the study of bubble tower phenomena. Our purpose is to give a constructive argument which enable us to give an estimate on how small ϵ should be, and to show that indeed ϵ needs not to be very small, e.g. We emphasize that in this context multiplicity results are very rare.

The proofs are based on some elementary tools of phase plane analysis.

The work is in collaboration with F. Dalbono from Palermo and A. Sfecci from Ancona.

[1] Chen C.C., Lin C.S., Blowing up with infinite energy of conformal metrics on S^n, Comm. Partial Differential Equations, **24** (1999), 785-799.

A spectral theorem for a fourth-order problem related to the dynamics of suspension bridges

Maurizio Garrione
Joint work with F. Gazzola (Polytechnic of Milan)

We study the fourth-order BVP

$$u'''' = \lambda u, \quad u(-\pi) = u(-a\pi) = u(a\pi) = u(\pi), \tag{1}$$

$a < 1$ being a positive parameter. Such a problem is related to the dynamics of suspension bridges, providing the fundamental modes of oscillation of a beam with two intermediate piers. We show that the presence of the piers produces some curious effects, among which the appearance of impulses which make the solutions of (1) *not regular*. Nonetheless, it is possible to prove the existence of a diverging sequence of simple (positive) eigenvalues and the corresponding eigenfunctions still enjoy suitable nodal properties, up to defining an appropriate associated notion of index. We will present a sketch of the proof and show some examples.

From Poincaré–Birkhoff Theorem to Maslov index: searching for more periodic solutions

Paolo Gidoni
CMAF-CIO Universidade de Lisboa

The Poincaré–Birkhoff Theorem and Maslov index are two classical tools to investigate existence and multiplicity of periodic solutions for Hamiltonian systems showing a twist behaviour. The relationship between these two approaches in the planar case was explained by Margheri, Rebelo and Zanolin in their 2002 paper. There, the authors also proved, using a modified Poincaré–Birkhoff Theorem, the existence of several T-periodic solutions for an asympotically linear planar Hamiltonian system, with different T-Maslov indices i_0 and i_∞ at zero and infinity. In particular, instead of the single T-periodic solution found by the general theory for $2N$-dimensional systems, they recovered $|i_\infty - i_0|$ solutions in most of the cases, and one solution less when i_0 is odd and i_∞ is even.

In this talk we first recall the main chapters of this story. Then, we show how, with an argument based on topological degree, this investigation can be completed, finding always $|i_\infty - i_0|$ periodic solutions, and sometimes up to a couple more.

Some analytical results about periodic orbits in the restricted three body problem with dissipation

Alessandro Margheri
Departamento de Matemática e CMAF-CIO
Faculdade de Ciências da Universidade de Lisboa

We present some analytical results about the existence of periodic orbits for the planar restricted three body problem with dissipation considered recently by Celletti et al. We show that, under fairly general conditions on the dissipation term, the circular orbits cannot be continued to the dissipative framework. Moreover, we give general conditions for the occurrence or not of a Hopf bifurcation around the libration points L_4 and L_5. Our results are consistent with the numerical findings of Celletti et al.

Resonance tongues in the linear Sitnikov equation

Mauricio Misquero
University of Granada

This research deals with a Hill's equation depending on two parameters, $e \in [0,1)$ and $\Lambda > 0$, that has applications to some problems in Celestial Mechanics of the Sitnikov-type. Due to the nonlinear dependence with respect to the eccentricity parameter e and the coexistence problem, the stability diagram in the (e,Λ)-plane presents unusual resonance tongues emerging from points $(0,(n/2)^2),\ n=1,2,...$ Some tongues vanishes whereas the remaining ones have no pockets and are very thin. Unlike most of the literature related to resonance tongues and Sitnikov-type problems, the study of these tongues is made from a global point of view in the whole range of $e \in [0,1)$. Indeed, it is found an interesting behavior of the tongues: almost all of them concentrate in a small Λ-interval $[1,9/8]$ as $e \to 1^-$. We apply the stability diagram of our equation to determine the regions for which the equilibrium of a Sitnikov $(N+1)$-body problem is stable in the sense of Lyapunov and the regions having symmetric periodic solutions with a given number of zeros.

The Poincaré - Birkhoff theorem and periodic solutions of second order ordinary differential equations

Carlota Rebelo
Departamento de Matemática e CMAF-CIO
Faculdade de Ciências da Universidade de Lisboa

We will recall the Poincaré-Birkhoff in the plane, describing a bit the story of its proof. Then we illustrate with a recent example its power in order to prove the existence of T-periodic solutions of second order equations.

Dynamics of predator prey models with a strong Allee effect on the prey and predator-dependent trophic functions

Cinzia Soresina
CMAF-CIO Universidade de Lisboa

The complex dynamics of a two-trophic chain are investigated. The chain is described by a general predator prey system, in which the prey growth rate and the trophic interaction functions are defined only by some properties determining their shapes. To account for undercrowding phenomena, the prey growth function is assumed to model a strong Allee effect; to simulate the predator interference during the predation process, the trophic function is assumed predator-dependent. A stability analysis of the system is performed, using the predation efficiency and a measure of the predator interference as bifurcation parameters. The admissible scenarios are much richer than in the case of prey-dependent trophic functions, investigated in Buffoni et al. (2011). General conditions for the number of equilibria, for the existence and stability of extinction and coexistence equilibrium states are determined, and the bifurcations exhibited by the system are investigated. Numerical results illustrate the qualitative behaviours of the system, in particular the presence of limit cycles, of global bifurcations and of bistability situations.

[1] G. Buffoni, M. Groppi, C. Soresina, *Effects of prey over-undercrowding in predator prey systems with prey-dependent trophic functions*, Nonlinear Analysis: Real World Applications, 12 (5) (2011) 2871-2887.

[2] G. Buffoni, M. Groppi, C. Soresina, *Dynamics of predator-prey models with a strong Allee effect on the prey and predator-dependent trophic functions*, Nonlinear Analysis: Real World Applications, 30 (2016) 143-169.

Multiplicity of positive solutions for some indefinite problems

Elisa Sovrano
Department of Mathematics, Computer Science and Physics,
University of Udine, Italy
sovrano.elisa@spes.uniud.it

We study Neumann problems associated with $u'' + w(x)f(u) = 0$, where $\mathcal{I}$ is a bounded interval, $w\colon \mathcal{I} \to \mathbb{R}$ is sign-changing and $f\colon [0,1] \to \mathbb{R}$ satisfies $f(0) = 0 = f(1)$, $f(s) > 0$, for every $s \in]0,1[$. Looking first at the graph of the nonlinearity f and then at the shape of the weight w, we deal with the multiplicity of positive solutions u such that $0 < u(x) < 1$ for every $x \in \mathcal{I}$.

Firstly, we answer a conjecture appeared in [Y. Lou, T. Nagylaki, *J. Differential Equations.* 2002] that states whether a uniqueness result of positive solutions holds if $\int_{\mathcal{I}} w(x)\,dx < 0$, the function f is not concave and the map $s \mapsto f(s)/s$ is decreasing. Motivated by the results achieved in [E. S., F. Zanolin, *Rend. Istit. Mat. Univ. Trieste.* 2015], we show the existence of at least 3 solutions of the Neumann problem considering a function f which fulfills the conjecture's conditions and has a strict local minimum in $]0,1[$, cf. [E. S., *J. Math. Biol.* 2017].

Secondly, taking into account [G. Feltrin, E. S., *Nonlinear Anal.* 2018], we prove the existence of at least 8 solutions of the Neumann problem, if w has two positive humps separated by a negative one and $f'(0) = 0$, cf. [G. Feltrin, E. S., *ArXiv* 2018].

Branches of forced oscillations induced by a delayed periodic force

Marco Spadini
University of Florence

In this talk I describe some recent progress in the understanding of the structure of the set of forced oscillations of constrained systems, induced by a periodic force that may depend on the whole (or on part of the) history of the system. Some applications to multiplicity results will be presented. This presentation is based upon results obtained in collaboration with Patrizia Pera and Alessandro Calamai that are, in turn, grounded on early work by Benevieri/Calamai/Furi/Pera, Furi/Pera, Furi/Pera/S. and Furi/S.

Favard condition and recurrent solutions of almost periodic equations

Massimo Tarallo
University of Milan

It is well known that linear almost periodic equations may not have almost periodic solutions, even when bounded solutions are known to exist. This pathology is related to the failure of the so–called Favard separation condition which, to some extent, is a kind of natural divide between nice and ugly equations. On the other hand, all the known examples of pathological equations admit solutions with a recurrence property weaker than almost periodicity: the so–called almost automorphy. During the talk, I will introduce the notion of almost automorphy and present a joint work with J. Campos, showing that such solutions exist in every almost periodic linear equations with bounded solutions. The result depends on a careful analysis of the way Favard separation condition breaks down.

The Ambrosetti-Prodi periodic problem: classical results and recent development.

Fabio Zanolin
University of Udine

We present recent results on the Ambrosetti-Prodi problem for nonlinear equations with periodic boundary conditions. In particular, we improve some classical assumptions of uniform coercivity to nonlocal ones and give some new applications to the periodically forced Liénard equation. We also discuss the presence of subharmonic solutions and complex dynamics, using different techniques.